Smart Grids Technologies
Solutions
Per-Unit Calculus

1 Linear Feeder

Definition of the Per-Unit Base

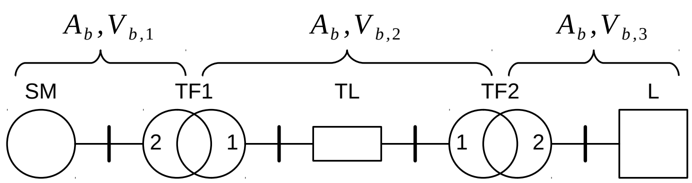

Figure 1: Per-unit bases.

The transformers divide the system into three subsystems (see Fig. 1). The nominal voltages of the transformers can serve as base voltages. So

$$V_{b,1} = 10 \text{ kV} \tag{1}$$
$$V_{b,2} = 220 \text{ kV} \tag{2}$$
$$V_{b,3} = 20 \text{ kV} \tag{3}$$

The base power can be chosen arbitrarily, for instance the nominal power of the first transformer. That is

$$A_b = 60 \text{ MVA} \tag{4}$$

Construction of the Per-Unit Equivalent Circuit

The per-unit equivalent circuit of the system is shown in Fig. 2. The output impedance of the synchronous machine is given w.r.t. the base $V_b' = V_n$,

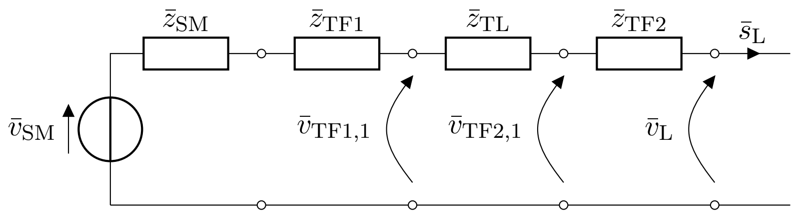

Figure 2: Per-unit equivalent circuit.

$A'_b = A_n$ (i.e., the nominal values of the synchronous machine). Therefore, a change of base is required:

$$\bar{z}_{\text{SM}} = jx_{\text{SM}} = j1.1\frac{A_b}{A'_b}\left(\frac{V'_b}{V_b}\right)^2 = j1.1\frac{60}{50}\left(\frac{12}{10}\right)^2 \approx j1.901 \tag{5}$$

The short-circuit impedance of the first transformer is

$$z_{sc,1} = V_{sc,1,\%} = \frac{10}{100} = 0.1 \tag{6}$$

$$\bar{z}_{\text{TF1}} = \bar{z}_{sc,1} = jx_{sc,1} \approx jz_{sc,1} = j0.1 \tag{7}$$

The series impedance of the transmission line is obtained as

$$\bar{z}_{\text{TL}} = jx_{\text{TL}} = jX_{\text{TL}} \cdot \frac{A_b}{V_b^2} = j65\frac{60}{220^2} \approx 0.081 \tag{8}$$

The short-circuit impedance of the second transformer is

$$z_{sc,2} = V_{sc,2,\%} \cdot \frac{A_b}{A_n} = \frac{10}{100}\frac{60}{30} = 0.2 \tag{9}$$

$$\bar{z}_{\text{TF2}} = \bar{z}_{sc,2} = jx_{sc,2} \approx jz_{sc,2} = j0.2 \tag{10}$$

Note that the nominal power of the second transformer is different from the base power. This is the reason for the correction factor above. Finally, the load power and voltage are

$$\bar{s}_{\text{L}} = \frac{24 + j15}{60} = 0.4 + j0.25 \tag{11}$$

$$v_{\text{L}} = \frac{19}{20} = 0.95 \tag{12}$$

Solution of the Per-Unit Network Equations

Without loss of generality, the phase angles of the voltages can be expressed relative to the load bus. Thus

$$\bar{v}_{\mathrm{L}} = v_{\mathrm{L}}\angle 0 = 0.95 \tag{13}$$

The load current is obtained as

$$\bar{i}_{\mathrm{L}} = \frac{\bar{s}_{\mathrm{L}}^*}{\bar{v}_{\mathrm{L}}^*} = \frac{0.4 - j0.25}{0.95} \approx 0.421 - j0.263 \tag{14}$$

The voltages can be computed straightforward

$$\begin{aligned}
\bar{v}_{\mathrm{SM}} &= \bar{v}_{\mathrm{L}} + (\bar{z}_{\mathrm{SM}} + \bar{z}_{\mathrm{TF1}} + \bar{z}_{\mathrm{TF2}} + \bar{z}_{\mathrm{TL}})\bar{i}_{\mathrm{L}} && (15)\\
&\approx 0.95 + j(1.901 + 0.1 + 0.2 + 0.081)(0.421 - j0.263) && (16)\\
&\approx 1.550 + j0.961 && (17)\\
\bar{v}_{\mathrm{TF1,1}} &= \bar{v}_{\mathrm{L}} + (\bar{z}_{\mathrm{TF2}} + \bar{z}_{\mathrm{TL}})\bar{i}_{\mathrm{L}} && (18)\\
&\approx 0.95 + j(0.2 + 0.081)(0.421 - j0.263) && (19)\\
&\approx 1.024 + j0.118 && (20)\\
\bar{v}_{\mathrm{TF2,1}} &= \bar{v}_{\mathrm{L}} + \bar{z}_{\mathrm{TF2}}\bar{i}_{\mathrm{L}} && (21)\\
&\approx 0.95 + j0.2(0.421 - j0.263) && (22)\\
&\approx 1.003 + j0.084 && (23)
\end{aligned}$$

Transformation from Relative to Absolute Units

The corresponding voltage magnitudes in absolute units are

$$\begin{aligned}
V_{\mathrm{SM}} &= |\bar{v}_{\mathrm{SM}}|\, V_{b,1} \approx 1.824 \cdot 10 \text{ kV} \approx 18.24 \text{ kV} && (24)\\
V_{\mathrm{TF1,1}} &= |\bar{v}_{\mathrm{TF1,1}}|\, V_{b,2} \approx 1.031 \cdot 220 \text{ kV} \approx 226.8 \text{ kV} && (25)\\
V_{\mathrm{TF2,1}} &= |\bar{v}_{\mathrm{TF2,1}}|\, V_{b,2} \approx 1.007 \cdot 220 \text{ kV} \approx 221.4 \text{ kV} && (26)
\end{aligned}$$

2 Parallel Transformers

Definition of the Per-Unit Base

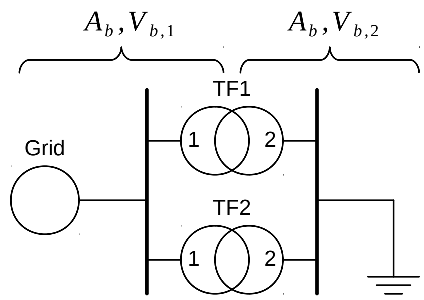

Figure 3: Per-unit bases.

The transformers divide the system into two subsystem (see Fig. 3). Again, take the nominal voltages of the transformers as base voltages. Namely

$$V_{b,1} = 20 \text{ kV} \tag{27}$$

$$V_{b,2} = 0.4 \text{ kV} \tag{28}$$

The base power can be chosen arbitrarily, for instance the nominal power of the transformer with the higher power rating. That is

$$A_b = 1000 \text{ kVA} \tag{29}$$

Construction of the Per-Unit Equivalent Circuit

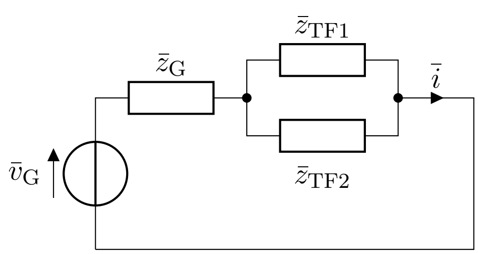

Figure 4: Per-unit equivalent circuit.

The per-unit equivalent circuit of the system is shown in Fig. 4. Without loss of generality, it can be assumed that the phase angle of the equivalent voltage source is zero. Thus

$$\bar{v}_{\mathrm{G}} = v_{\mathrm{G}}\angle 0 = \frac{V_n}{V_b} = \frac{20}{20} = 1 \tag{30}$$

Since the short-circuit impedance is assumed to be purely inductive

$$z_{\mathrm{G}} = z_{sc} = \frac{Z_{sc}}{Z_b} = \frac{V_n^2}{S_{sc}}\frac{S_b}{V_{b,1}^2} = \frac{1}{348}\frac{20^2}{20^2} \approx 0.00287 \tag{31}$$

$$\bar{z}_{\mathrm{G}} \approx jx_{\mathrm{G}} = jz_{\mathrm{G}} \approx j0.002874 \tag{32}$$

The short-circuit impedances of the transformers are obtained as follows

$$z_{sc,1} = V_{sc,\%} = \frac{5}{100} = 0.05 \tag{33}$$

$$\bar{z}_{\mathrm{TF1}} = z_{sc,1}(\cos\phi_{sc} + j\sin\phi_{sc}) = 0.05 \cdot (0.22 + j\sqrt{1-0.22^2}) \tag{34}$$

$$z_{sc,2} = V_{sc,\%} \cdot \frac{A_b}{A_n} = \frac{5}{100}\frac{1000}{400} = 0.125 \tag{35}$$

$$\bar{z}_{\mathrm{TF2}} = z_{sc,2}(\cos\phi_{sc} + j\sin\phi_{sc}) = 0.125 \cdot (0.22 + j\sqrt{1-0.22^2}) \tag{36}$$

The parallel combination of the two impedances yields

$$\bar{z}_{\mathrm{TF}} = (\bar{z}_{\mathrm{TF1}}^{-1} + \bar{z}_{\mathrm{TF2}}^{-1})^{-1} = \frac{0.05 \cdot 0.124}{0.05 + 0.125}(0.22 + j\sqrt{1-0.22^2}) \tag{37}$$

$$\approx 0.00786 + j0.03484 \tag{38}$$

Solution of the Per-Unit Network Equations

The short-circuit current is straightforwardly obtained as

$$\bar{i} = \frac{\bar{v}_{\mathrm{G}}}{\bar{z}_{\mathrm{G}} + \bar{z}_{\mathrm{TF}}} \approx \frac{1}{0.0078 + j0.0378} \approx 5.295 - j25.413 \tag{39}$$

with a magnitude of

$$i = |\bar{i}| = |5.295 - j25.413| \approx 25.96 \tag{40}$$

Transformation from Relative to Absolute Units

The corresponding current magnitude in absolute units is

$$I_2 = i \cdot I_{b,2} = i\frac{A_b}{\sqrt{3}V_{b,2}} \approx 25.96\frac{1000}{1.732 \cdot 0.4} \approx 37.5\ \mathrm{kA} \tag{41}$$